

---

## Contents

Preface .....	vii
---------------	-----

---

### Part I Gaussian Processes

---

<b>1 Gaussian Fields</b> .....	7
1.1 Random Fields .....	7
1.2 Gaussian Variables and Fields .....	8
1.3 Boundedness and Continuity .....	11
1.4 Examples .....	20
1.4.1 Fields on $\mathbb{R}^N$ .....	20
1.4.2 Differentiability on $\mathbb{R}^N$ .....	22
1.4.3 The Brownian Family of Processes .....	24
1.4.4 Generalized Fields .....	30
1.4.5 Set-Indexed Processes .....	36
1.4.6 Non-Gaussian Processes .....	40
1.5 Majorizing Measures .....	41
<b>2 Gaussian Inequalities</b> .....	49
2.1 Borell–TIS Inequality .....	49
2.2 Comparison Inequalities .....	57
<b>3 Orthogonal Expansions</b> .....	65
3.1 The General Theory .....	66
3.2 The Karhunen–Loève Expansion .....	70
<b>4 Excursion Probabilities</b> .....	75
4.1 Entropy Bounds .....	76
4.2 Processes with a Unique Point of Maximal Variance .....	86
4.3 Examples .....	89
4.4 Extensions .....	93

4.5	The Double-Sum Method . . . . .	95
4.6	Local Maxima and Excursion Probabilities . . . . .	96
<b>5</b>	<b>Stationary Fields . . . . .</b>	<b>101</b>
5.1	Basic Stationarity . . . . .	101
5.2	Stochastic Integration . . . . .	103
5.3	Moving Averages . . . . .	105
5.4	Spectral Representations on $\mathbb{R}^N$ . . . . .	109
5.5	Spectral Moments . . . . .	112
5.6	Constant Variance . . . . .	114
5.7	Isotropy . . . . .	115
5.8	Stationarity over Groups . . . . .	119

**Part II Geometry**

<b>6</b>	<b>Integral Geometry . . . . .</b>	<b>127</b>
6.1	Basic Integral Geometry . . . . .	127
6.2	Excursion Sets Again . . . . .	134
6.3	Intrinsic Volumes . . . . .	141
<b>7</b>	<b>Differential Geometry . . . . .</b>	<b>149</b>
7.1	Manifolds . . . . .	149
7.2	Tensor Calculus . . . . .	154
7.3	Riemannian Manifolds . . . . .	160
7.4	Integration on Manifolds . . . . .	166
7.5	Curvature . . . . .	171
7.6	Intrinsic Volumes for Riemannian Manifolds . . . . .	175
7.7	A Euclidean Example . . . . .	176
<b>8</b>	<b>Piecewise Smooth Manifolds . . . . .</b>	<b>183</b>
8.1	Whitney Stratified Spaces . . . . .	184
8.2	Locally Convex Spaces . . . . .	188
8.3	Cone Spaces . . . . .	190
<b>9</b>	<b>Critical Point Theory . . . . .</b>	<b>193</b>
9.1	Critical Points . . . . .	193
9.2	The Normal Morse Index . . . . .	195
9.2.1	The Index . . . . .	195
9.2.2	Generalized Tangent Spaces and Tame Manifolds . . . . .	196
9.2.3	Regular Stratified Manifolds . . . . .	198
9.2.4	The Index on Intersections of Sets . . . . .	198
9.3	Morse's Theorem for Stratified Spaces . . . . .	206
9.3.1	Morse Functions . . . . .	206
9.3.2	Morse's Theorem . . . . .	207

9.4	The Euclidean Case .....	210
<b>10</b>	<b>Volume of Tubes .....</b>	<b>213</b>
10.1	The Volume-of-Tubes Problem .....	215
10.2	Volume of Tubes and Gaussian Processes .....	216
10.3	Local Geometry of Tube( $M, \rho$ ) .....	219
10.3.1	Basic Structure of Tubes .....	220
10.3.2	Stratifying the Tube .....	222
10.4	Computing the Volume of a Tube .....	223
10.4.1	First Steps .....	223
10.4.2	An Intermediate Computation .....	224
10.4.3	Subsets of $\mathbb{R}^l$ .....	225
10.4.4	Subsets of Spheres .....	230
10.5	Weyl's Tube Formula .....	231
10.6	Volume of Tubes and Gaussian Processes, Continued .....	242
10.7	Intrinsic Volumes for Whitney Stratified Spaces .....	244
10.7.1	Alternative Representation of the Curvature Measures .....	249
10.8	Breakdown of Weyl's Tube Formula .....	249
10.9	Generalized Lipschitz–Killing Curvature Measures .....	250
10.9.1	The Generalized Curvature Measures .....	251
10.9.2	Surface Measure on the Boundary of a Tube .....	252
10.9.3	Series Expansions for the Gaussian Measure of Tubes .....	254

### Part III The Geometry of Random Fields

<b>11</b>	<b>Random Fields on Euclidean Spaces .....</b>	<b>263</b>
11.1	Rice's Formula .....	263
11.2	An Expectation Metatheorem .....	266
11.3	Suitable Regularity and Morse Functions .....	280
11.4	An Alternate Proof of the Metatheorem .....	283
11.5	Higher Moments .....	284
11.6	Preliminary Gaussian Computations .....	286
11.7	The Mean Euler Characteristic .....	289
11.8	Mean Intrinsic Volumes .....	298
11.9	On the Importance of Stationarity .....	299
<b>12</b>	<b>Random Fields on Manifolds .....</b>	<b>301</b>
12.1	The Metatheorem on Manifolds .....	301
12.2	Riemannian Structure Induced by Gaussian Fields .....	305
12.2.1	Connections and Curvatures .....	306
12.2.2	Some Covariances .....	308
12.2.3	Gaussian Fields on $\mathbb{R}^N$ .....	310
12.3	Another Gaussian Computation .....	312
12.4	The Mean Euler Characteristic .....	315

12.4.1	Manifolds without Boundary .....	315
12.4.2	Manifolds with Boundary .....	317
12.5	Examples .....	323
12.6	Chern–Gauss–Bonnet Theorem .....	327
<b>13</b>	<b>Mean Intrinsic Volumes .....</b>	<b>331</b>
13.1	Crofton’s Formula .....	332
13.2	Mean Intrinsic Volumes: The Isotropic Case .....	333
13.3	A Gaussian Crofton Formula .....	334
13.4	Mean Intrinsic Volumes: The General Case .....	342
13.5	Two Gaussian Lemmas .....	343
<b>14</b>	<b>Excursion Probabilities for Smooth Fields .....</b>	<b>349</b>
14.1	On Global Suprema .....	351
14.1.1	A First Representation .....	352
14.1.2	The Problem with the First Representation .....	354
14.1.3	A Second Representation .....	354
14.1.4	Random Fields .....	360
14.1.5	Suprema and Euler Characteristics .....	362
14.2	Some Fine Tuning .....	365
14.3	Gaussian Fields with Constant Variance .....	368
14.4	Examples .....	372
14.4.1	Stationary Processes on $[0, T]$ .....	372
14.4.2	Isotropic Fields with Monotone Covariance .....	374
14.4.3	A Geometric Approach .....	376
14.4.4	The Cosine Field .....	382
<b>15</b>	<b>Non-Gaussian Geometry .....</b>	<b>387</b>
15.1	A Plan of Action .....	389
15.2	A Representation for Mean Intrinsic Volumes .....	391
15.3	Proof of the Representation .....	392
15.4	Poincaré’s Limit .....	398
15.5	Kinematic Fundamental Formulas .....	400
15.5.1	The KFF on $\mathbb{R}^n$ .....	401
15.5.2	The KFF on $S_\lambda(\mathbb{R}^n)$ .....	402
15.6	A Model Process on the $l$ -Sphere .....	402
15.6.1	The Process .....	403
15.6.2	Mean Curvatures for the Model Process .....	404
15.7	The Canonical Gaussian Field on the $l$ -Sphere .....	410
15.7.1	Mean Curvatures for Excursion Sets .....	411
15.7.2	Implications for More General Fields .....	415
15.8	Warped Products of Riemannian Manifolds .....	416
15.8.1	Warped Products .....	417
15.8.2	A Second Fundamental Form .....	419
15.9	Non-Gaussian Mean Intrinsic Volumes .....	421

15.10 Examples .....	425
15.10.1 The Gaussian Case .....	426
15.10.2 The $\chi^2$ Case .....	427
15.10.3 The $F$ Case .....	430
<b>References .....</b>	<b>435</b>
<b>Notation Index .....</b>	<b>443</b>
<b>Subject Index .....</b>	<b>445</b>